

**COMPUTATION OF THE TEST STATISTIC AND THE NULL
DISTRIBUTION IN THE MULTIVARIATE ANALOGUE OF
THE ONE-SIDED TEST**

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Abbreviated title: Computation of the Test Statistic and the Null Distribution

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ABSTRACT

The multivariate analogue of the one sided test derived in Kudô (1963) is considered. A handy method of computing the test statistic and its significance probability is given. The method is based on applying sweep out operations on a certain matrix in a systematic manner and applying the Fortran subroutine of Sun (1988).

1. Introduction

The problem of this paper is as follows. Given a p -variate normal distribution: $N_p(\boldsymbol{\theta}, \mathbf{\Lambda})$, we consider the testing problem: $H_0 : \boldsymbol{\theta} = \mathbf{0}$ versus $H_1 - H_0 : \boldsymbol{\theta} \geq \mathbf{0}$, where $\mathbf{\Lambda}$ is assumed to be known and $\boldsymbol{\theta} = (\theta_1, \dots, \theta_p)' \geq \mathbf{0}$ means $\text{Max } \theta_i > 0, \text{Min } \theta_i \geq 0$, thus the space of the alternative hypothesis is a cone. The MLE of $\boldsymbol{\theta}$ in the space of $H_0 \cup H_1$ is often called “the projection into the cone” because of the very nature of the alternative hypothesis. In this paper we also set the hypothesis $H_2 : \boldsymbol{\theta} \neq \mathbf{0}$, and consider a testing problem : H_1 versus $H_2 - H_1$.

More than 3 decades have passed since the paper “Multivariate Analogue of the One-sided Test” was published in *Biometrika* where the above problem was treated and the likelihood ratio test was derived. This paper has been criticized in two aspects. Difficulties in computing (i) the test statistic and (ii) the significance probability. (See for instance Tang *et al.* (1989). See also Barlow *et al.* (1972) and Robertson *et al.* (1988).)

Our purpose is to demonstrate methods to compute these two. In Section 2 we quickly review the outline of the problem, and Section 3 is devoted to the method for computing the MLE, which will find its application in computing an optimum linear test given in Shi *et al.* (1987), and Section 4 to a numerical example. In Section 5 an application of Sun’s subroutine is demonstrated and its accuracy examined, and the final Section is for concluding discussions.

The first of two distribution functions given in Section 2 was given by Kudô (1963) and the second can be derived from Theorem 2.3.1. of Robertson *et al.* (1988). The second distribution was treated in Robertson and Wegman (1978) and we documented the same in the present paper’s form.

1.1. How to use the .cls or .sty files

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We already defined some environment for theorems, propositions, *etc.* See this T_EX source file and the following examples.

Lemma 1 *This is an example of the lemma environment.*

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}. \quad (1)$$

Theorem 1 *This is an example of the theorem environment.*

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}. \quad (2)$$

In theorem environment, the reference to an equation(s) should not be italicized. Use \rm command (L^AT_EX 2.09) or \textup{} command (L^AT_EX 2_ε) like “From the equation (1), ...”, “According to the conditions (i)–(iv), ...”, etc.

Proof. In our .cls and .sty files, \proof command and \qed command are defined. You can use \proof for opening the proof and \qed for closing the proof. □

2. A brief review

As the variance matrix Λ is assumed to be known, we can assume there is only one observation x . Let P be the set of integers $P = \{1, \dots, p\}$ and $M = \{i_1, \dots, i_m\}$ be a subset

of P . It was shown that there exists uniquely a set satisfying the condition stated below. (see Beale (1959))

Without loss of generality, we state the condition in the case when the set is the first m natural numbers: $M = \{1, \dots, m\}$ where $m \leq p$.

Let the observation vector $\mathbf{x}' = (x_1, \dots, x_p)$ be partitioned into m and $p-m$ components and the mean vector and the variance matrix be partitioned accordingly:

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_{(1)} \\ \mathbf{x}_{(2)} \end{pmatrix}, \quad \boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_{(1)} \\ \boldsymbol{\theta}_{(2)} \end{pmatrix}, \quad \boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} \\ \boldsymbol{\Lambda}_{21} & \boldsymbol{\Lambda}_{22} \end{pmatrix}.$$

There exists uniquely the subset M satisfying the condition.

$$\begin{cases} \boldsymbol{\Lambda}_{11}^{-1} \mathbf{x}_{(1)} < 0 \\ \mathbf{x}_{(2)} - \boldsymbol{\Lambda}_{21} \boldsymbol{\Lambda}_{11}^{-1} \mathbf{x}_{(1)} \geq 0 \end{cases} \quad (3)$$

When $m = 0$, $M = \phi$, the empty set, and when $m = p$, the second condition in (3) is vacuous.

Table 1 is inserted here.

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Table 1: The first $2^p - 1$ binary numbers

No.	Binary No.	Sweep out position
1	1	1000...0
2	10	0100...0
3	11	1100...0
4	100	0010...0
\vdots	\vdots	\vdots
2^{p-2}	$1\dots 1$	$111\dots 10$
$2^{p-2} + 1$	$10\dots 0$	$000\dots 01$
\vdots	\vdots	\vdots
2^{p-1}	$11\dots 1$	$111\dots 11$